

# Optimal Transmission Expansion Planning Using Ant Colony Optimization

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**Abstract:** This paper proposes the application of ant colony optimization (ACO) to solve a static transmission expansion planning (STEP) problem based on a DC power flow model. The major objective is to minimize the investment cost of transmission lines added to an existing network in order to supply the forecasted load as economically as possible and subject to many system constraints i.e. the power balance, the generation requirements, line connections and thermal limits. The Garver's six-buses system, is analyzed to appraise the feasibility of the ACO. The experimental results obtained by ACO are compared to those obtained by the conventional approaches of the Genetic Algorithm (GA), and the Tabu Search (TS) algorithm. The results show that the ACO method outperforms other methods in convergence characteristic and computational efficiency.

**Keywords:** Ant colony optimization, Genetic algorithm, Power system planning, Tabu Search algorithm, and Transmission Expansion Planning.

## 1. Introduction

Transmission expansion planning (TEP) is an important issue in electrical power system planning. The principle of TEP is to find an optimal configuration to be consistent with the electricity demand and a generation planning scheme, meeting the requirement of delivering sufficient electricity, safely and economically over the planning period. For that reason, TEP addresses an optimization problem in power systems. Models of TEP can be categorized as static or dynamic according to the treatment of the study period. Static planning involves single horizontal planning and answers the questions of *what* type of and *where* new equipment should be installed in a way that minimizes the installation and operational costs. Dynamic, or multiple stage planning, is a derived generalization that considers the separation of the planning horizon into several stages and answers the questions of *what*, *where*, and *when* to install the network additions [1]. This paper focuses only on static planning.

In the past decade, many researchers proposed various techniques to solve both static and dynamic planning problems. In the literatures, mathematical models applied to TEP were classified exclusively by Latorre et al. [2]. A classical Linear Programming (LP) algorithm to solve static TEP was proposed by Garver [3] and Villasana et al. [4]. For dynamic planning, Escobar et al. [5] proposed an efficient genetic algorithm (GA) to solve the problem of multistage and coordinated TEP. Many artificial intelligence (AI) methods have been applied for solving TEP problems. These methods include Simulated Annealing (SA) [6], Genetic Algorithm (GA) [7], Tabu Search (TS) [8], Particle Swarm Optimization (PSO) [9] and Differential Evolution (DE) algorithm [10]. Only recently, Ant Colony Optimization (ACO) has become a candidate for many potential applications. The ACO algorithm is inspired by the behavior of real ants and was first introduced by Marco Dorigo et al. to solve the Traveling Salesman Problem (TSP) [11], and was later applied to the asymmetric TSP [12], the Quadratic Assignment Problem (QAP) [13] and the Vehicle Routing Problem (VRP) [14]. Recently, ACO has been adapted to some network problems e.g. power distribution expansion planning [15], optimal placement of switches and protective devices in electric power distribution systems [16] and communication network design [17].

However, nothing in the literatures mentions an application of ACO to TEP in the power transmission domain. It is envisaged that the ACO method applies very well to searching for the shortest path and to solving discrete problems such as unit commitment (UC) and transmission expansion models. On the contrary, the method does not apply well for continuous problems such as

economic dispatch (ED) or optimal power flow (OPF). Since the TEP problem is formulated as an integer programming optimization (discrete variable), the ACO method applies particularly well to this problem.

This paper introduces the application of ACO to the DC power flow-based model for solving the static TEP. The results obtained by ACO are compared to those obtained by conventional approaches i.e. Genetic Algorithm (GA) and Tabu Search (TS) methods in term of solution quality, convergence characteristic and computational efficiency.

The remaining part of this paper is organized as follows: Section 2 formulates TEP problem based on a DC load flow model. Section 3 elaborates the principle of the ACO algorithm. Section 4 presents detailed procedures for the ACO approach to solving the static TEP problem. Section 5 gives a case study and compares the results with those of the traditional methods. Section 6 presents the conclusion of the study.

## 2. Methodology

### 2.1 Problem formulation

#### 2.1.1 Objective Function of TEP

The objective function of TEP is to minimize the investment cost of transmission lines subjected to physical and economic constraints. In this paper, the classical DC power flow model is used for static TEP, which can be formulated as [18]:

$$\text{minimize } C_T = \sum_{i,j \in \Omega} c_{ij} n_{ij} \quad (1)$$

where

- $C_T$  is the total investment cost of transmission lines.
- $c_{ij}$  is the cost of a circuit to be added to the right-of-way  $i-j$ .
- $n_{ij}$  is the number of circuits added to the right-of-way  $i-j$ .
- $\Omega$  is the set of all rights-of-way  $i-j$ .

#### 2.1.2 Constraints

##### 2.1.2.1 Equality constraints.

According to the set of equations determined by Kirchoff's laws (KCL, KVL), the power flows in the system are expressed as:

##### (a) Power Balance

This constraint represents the conservation of power in each node.

$$SP_i + P_g - P_d = 0 \quad (2)$$

Based on the above assumptions, the DC load flow can be obtained by the following equation.

$$P_i = \sum_{j=1}^N (B_{ij}(\theta_i - \theta_j)) \quad ; \quad i = 1,2,3,\dots,N \quad (3)$$

where

- $S$  is the branch-node incidence transposed matrix.
- $P_i$  is the real power flow injection at bus  $i$ .
- $P_g$  is generation injection of nodes (generation in bus  $k$ ).
- $P_d$  is the load demand vector in all networks nodes.
- $B_{ij}$  is the susceptance matrix of the existing and added lines in the network
- $N$  is the total number of buses in the system.

**(b) Kirchoff's voltage law (KVL)**

This law is the conservation of energy in the equivalent DC network Subject to constraints that are nonlinear.

$$P_{ij} - b_{ij}(n_{ij}^0 + n_{ij})(\theta_i - \theta_j) = 0 \quad (4)$$

where

- $P_{ij}$  is the power flow in branch  $i - j$ .
- $b_{ij}$  is the susceptance in the right-of-way  $i-j$ .
- $n_{ij}^0$  is the number of circuits in the original base system
- $\theta_i, \theta_j$  is the phase angle of the terminal bus  $i$  and  $j$ .

**2.1.2.2 Inequality constraints**

The inequality constraints reflect the limits on physical and economic devices in the power system as well as the limits created to ensure system security.

**(a) Transmission capacity limit or power flow limit.**

These constraints can represent the maximum power of the transmission line that can be carried based on thermal and dynamic stability considerations.

$$|P_{ij}| \leq (n_{ij}^0 + n_{ij}) \cdot P_{ij}^{\max} \quad (5)$$

where

- $P_{ij}^{\max}$  is the maximum branch power flow in the right-of-way  $i-j$ .

**(b) Power generating limit.**

These constraints give the maximum and minimum generating capacities, outside of which it is not feasible to generate power due to technical or economic reasons.

$$P_g^{\min} \leq P_g \leq P_g^{\max} \quad (6)$$

where

- $P_g^{\min}$  is the minimum active power output generated at bus  $k$ .
- $P_g^{\max}$  is the maximum active power output generated at bus  $k$ .

**(c) Right-of-way limit.**

For transmission planning, planners need to know the exact location and capacity of the new required lines. Therefore, this constraint has to be included for consideration in the planning. Mathematically, this constraint defines the line location and the maximum number of lines that can be installed in a specified location. It is represented as follow:

$$0 \leq n_{ij} \leq n_{ij}^{\max} \quad (7)$$

where

- $n_{ij}^{\max}$  is the maximum number of circuits that can be added in the right-of-way  $i-j$ .

**2.2 Ant colony optimization**

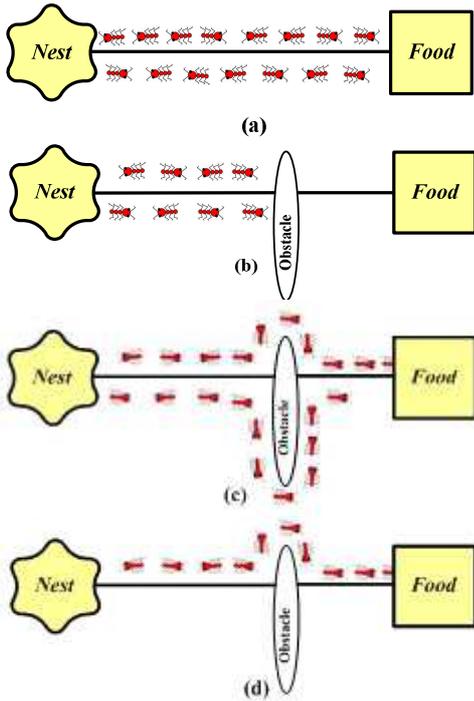
Inspired by the collective behavior of a real ant colony, Marco Dorigo first introduced the Ant System (AS) in his Ph.D. thesis in 1992, and published more about it in [19]. The characteristics of an artificial ant colony include positive feedback, distributed computation, and the use of a constructive greedy heuristic. Positive feedback accounts for rapid discovery of good solutions, distributed computation avoids premature convergence, and the greedy heuristic helps to find acceptable solutions in the early stages of the search process. In order to demonstrate the AS approach, the authors applied this approach to the classical TSP, asymmetric TSP, Quadratic Assignment Problem (QAP), Job-Shop Scheduling Problem (JSSP) and Vehicle Routing Problem (VRP). The AS shows very good results in each applied area. More recently, Marco Dorigo and Gambardella [11] have been working on extended versions of the AS paradigm. ACO is one of the extensions and has been applied to both symmetric and asymmetric TSP with excellent results in literature. The ant system approach has also been applied successfully to other combinatorial optimization problems, such as the classical TSP, asymmetric TSP, quadratic assignment problem and the vehicle routing problem.

ACO is an algorithm which was inspired by the behavior of real ants [11]. Entomologists have studied how blind animals, such as ants, are capable of finding the shortest path from food sources to the nest without using visual cues. They are also capable of adapting themselves to a changing environment. For example, finding a new shortest path once the old one is no longer feasible due to a new obstacle. The studies by entomologists reveal that such capabilities are essentially due to communicating information about the paths among individuals to decide the best direction. Ants deposit a certain amount of pheromones while walking, and each ant probabilistically prefers to follow a direction rich in pheromones rather than a poorer one.

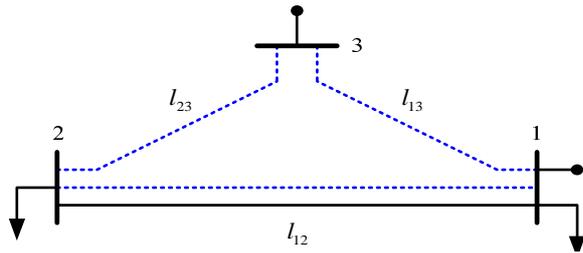
Fig.1 illustrates the searching behavior of an ant colony In Fig.1(a) ants are on a straight line that connects a food source to their nest. An ant will deposit pheromone while walking and it probabilistically prefers to follow a direction rich in pheromone. In Fig. 1(b), the ants are obstructed and cannot continue in a straight line. Therefore, they have to choose between turning right or left. Half of the ants choose to turn right and the other half choose to turn left. In Fig. 1(c), ants choosing the shorter path will more rapidly reconstitute the interrupted pheromone trail than those choosing the longer route. Thus, the shorter path will receive a greater amount of pheromones per time unit and, so, larger numbers of ants will choose the shorter path. Due to this positive feedback, eventually, all the ants will choose the shorter path as shown in Fig. 1(d). Each ant moves at approximately the same speed and deposits a pheromone trail at approximately the same rate. The time consumed on the longer side of an obstacle is greater than the shorter one, making the pheromone trail accumulate more quickly on the shorter side. Ants prefer higher pheromone trail levels and will cause the accumulation to build up faster on the shorter route.

**2.3 Implementation of ACO for TEP**

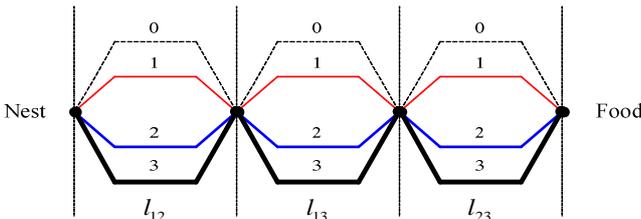
This section elaborates an application of the ACO algorithm to solve TEP. First, the model has to be formulated as in Fig. 1 with routes between a nest and food source. For example, in a system consisting of 3 buses and 3 branches, each branch can have a number of possible lines and possible rights-of-way as shown in Fig. 2. This network can be graphically translated as the radial routes between the nest and the food source as shown in Fig. 3. This model reveals that the transmission system topology can be constructed by randomly selecting a number of line(s) for each branch, analogous to the ant's route between the nest and the food source.



**Figure 1.** Behavior of ants.  
 (a) Real ants follow a path between nest and food source.  
 (b) An obstacle appears on the path: ants choose whether to turn left or right with equal probability.  
 (c) Pheromone is deposited more quickly on the shorter path.  
 (d) All ants have chosen the shorter path.

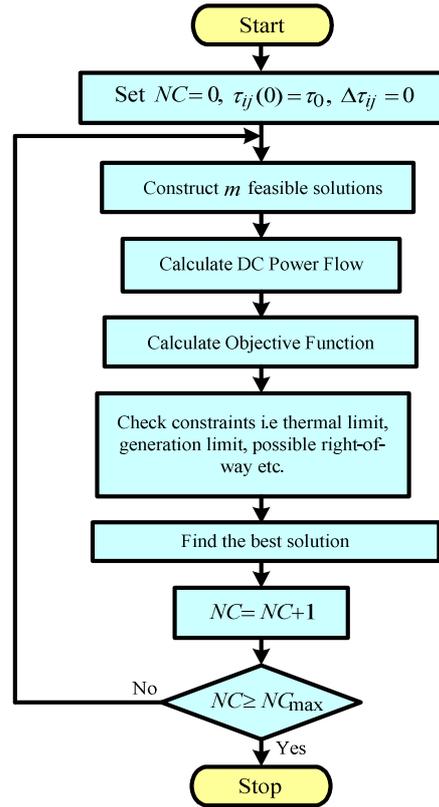


**Figure 2.** Example of a power system with 3 buses 3 branches.



**Figure 3.** A simulation model of the routes of ants between the nest and food source.

In general, the procedure of an ACO algorithm can be described as follows: an  $m$  number of ants are initially positioned at the nest. Each ant will choose a possible route as a solution. In fact, each ant builds a feasible solution (called a tour) by repeatedly applying a stochastic greedy search, called, the state transition rule. Once all ants have terminated their tours, the following steps are performed: the amount of pheromone is modified by applying the global updating rule [14]. Ants are guided, in building their tours by both heuristic and pheromone information. Naturally, a link with a high amount of pheromone is a desirable choice. The pheromone updating rules are designed so that they tend to give more pheromones to the edges, which should be visited by ants. A flowchart of the proposed TEP-ACO and its algorithm is shown in Fig. 4. The details of ACO algorithm can be described in the following steps.



**Figure 4.** Flow chart of TEP-ACO algorithm.

**Step 1 Initialization**

Set  $NC = 0$  /\* NC: Cycle Counter \*/  
 For every combination  $(i, j)$   
 Set an initial value  $\tau_{ij}(0) = \tau_0$  and  $\Delta\tau_{ij} = 0$

End

**Step 2 Construct feasible solutions**

For  $k=1$  to  $m$ /\*  $m$ : number of ants \*/  
 For  $i=1$  to  $n$  /\*  $n$ : number of branches \*/  
 Choose a level of connection with transition probability given by Eq. (10).  
 End  
 Calculate Objective Function Eq. (1) and Check Constraints Eq. (2-7)  
 End

Update the best solution.

**Step 3 Global updating rule**

For every combination  $(i, j)$   
 For  $k=1$  to  $m$   
 Find  $\Delta\tau_{ij}^k$  according to Eq. (12)  
 End  
 Update  $\Delta\tau_{ij}$  according to Eq. (11).

End

Update the trail values according to Eq. (10).  
 Update the transition probability according to Eq. (8).

**Step 4 Next search**

Set  $NC = NC+1$   
 For every combination  $(i, j)$   
 $\Delta\tau_{ij} = 0$   
 End

**Step 5 Termination**

If  $(NC < NC_{max})$   
 Then  
 Go to step 2, Else  
 Print the best feasible solution  
 Stop  
 End

**(A) State transition rule**

The state transition rule of the ant colony is given in Eq. (8). This equation represents the probability that an ant  $k$  selects a link that connects bus  $i$  to bus  $j$  :

$$p_{ij}^k(t) = \frac{[\tau_{ij}(t)]^\alpha [\eta_{ij}(t)]^\beta}{\sum_{k=1}^m [\tau_{im}(t)]^\alpha [\eta_{im}(t)]^\beta} \quad (8)$$

where

- $\tau_{ij}$  is the pheromone intensity.
- $\eta_{ij}$  is the heuristic information between node  $i$  and node  $j$ .
- $\alpha$  is the relative importance of the trail.
- $\beta$  is the relative importance of the heuristic information.

The problem specific heuristic information is:

$$\eta_{ij} = \frac{1}{c_{ij}} \quad (9)$$

where

$c_{ij}$  is the associated cost of the transmission lines.

Therefore, the level of line with less cost has a greater probability of being chosen.

**(B) Global updating rule**

During the construction of the solution, there is no guarantee that an ant will construct a feasible solution that obeys the reliability constraint. The pheromone updating remedies an infeasible solution. The amount of deposited pheromones is set to a high value if the generated solution is feasible and to a low value if it is infeasible. Therefore, this value depends on the solution quality. Infeasibility can be handled by assigning a penalty in proportion to the amount of reliability violations. In the case of a feasible solution, an additional penalty is introduced to improve its quality.

Following the above remarks, the trail intensity is updated as follows:

$$\tau_{ij}(t) = (1 - \rho) \cdot \tau_{ij}(t - 1) + \Delta \tau_{ij} \quad (10)$$

where

$\rho$  is a coefficient such that  $(1 - \rho)$  represents the evaporation of a trail.

and  $\Delta \tau_{ij}$  is:

$$\Delta \tau_{ij} = \sum_{k=1}^m \Delta \tau_{ij}^k \quad (11)$$

where

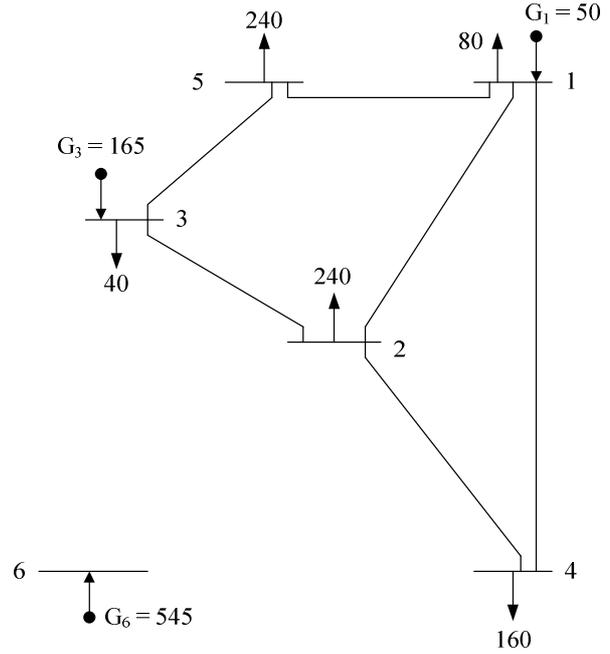
$m$  is the number of ants.

and  $\Delta \tau_{ij}^k$  is given by:

$$\Delta \tau_{ij}^k = \begin{cases} 1 & \text{if } k^{\text{th}} \text{ ant chooses path} \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

**3. Results and Discussion**

The proposed TEP-ACO method was tested on the well-known Garver's 6-bus test system shown in Fig. 5. The Garver system has 6 buses and 15 candidate branches. The total demand is 760 MW and the relevant data are given in Tables 1 and 2. The maximum possible number of added lines (right-of-way limit) per branch equals four.



**Figure 5.** Initial configuration of Garver's 6-bus network.

**Table 1.** Generation and load data for Garver's 6-bus system.

Bus	Generation (MW)		Demand (MW)
	Max.	Level	
1	150	50	80
2	-	-	240
3	360	165	40
4	-	-	160
5	-	-	240
6	600	545	-

**Table 2.** Branch data for Garver's 6-bus system.

From-To ( $\Omega$ )	$n_{ij}^0$	$r$ (p.u)	$x$ (p.u)	$P_{ij}^{\max}$	Cost ( $\times 10^3$ US\$)
1-2	1	0.10	0.40	100	40
1-3	0	0.09	0.38	100	38
1-4	1	0.15	0.60	80	60
1-5	1	0.05	0.20	100	20
1-6	0	0.17	0.68	70	68
2-3	1	0.05	0.20	100	20
2-4	1	0.10	0.40	100	40
2-5	0	0.08	0.31	100	31
2-6	0	0.01875	0.30	100	30
3-4	0	0.15	0.59	82	59
3-5	1	0.25	0.20	100	20
3-6	0	0.12	0.48	100	48
4-5	0	0.16	0.63	75	63
4-6	0	0.0375	0.30	100	30
5-6	0	0.15	0.61	78	61

The optimal planning solution for Garver's system is  $n_{26} = 4$ ,  $n_{35} = 1$ , and  $n_{46} = 2$  as shown in Fig. 6. The simulation was made for comparison to the GA and TS approaches. All methods were performed for 30 trials, under the same evaluation function and individual definition, in order to compare their solution quality, convergence characteristic, and computational efficiency. The programs were implemented by

MatLab® languages on an Intel® Core2 Duo 1.66 GHz Laptop with 2 GB RAM under Windows XP. The statistical results, such as the investment cost, standard deviation, computational time and percentage of approaching near optimal solution, are shown in Table 3.

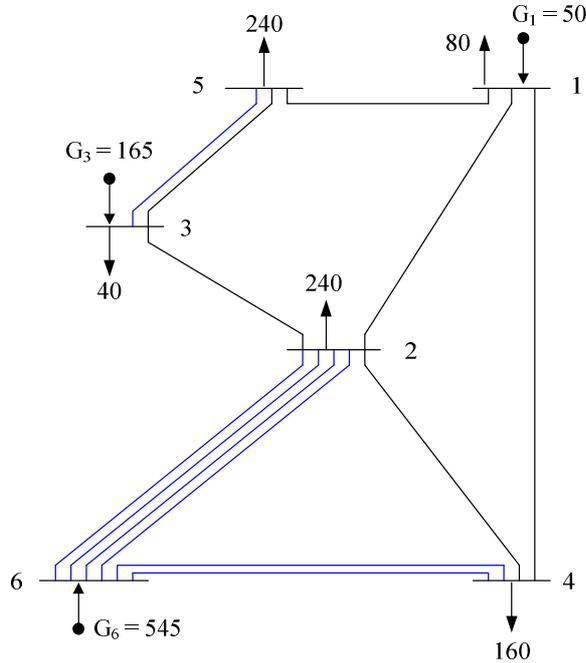


Figure 6. Optimal plan of Garver's 6-bus system.

Table 3. Results of Garver's 6-bus Test System.

Methods	$n_{ij}$	Investment Cost (10 <sup>3</sup> US\$)			SD	% Get Optimum Cost	Used Time
		Worst	Average	Best			
GA	7	368	227	200	41.27	53.333	46.686
TS	7	244	218	200	26.56	86.776	36.983
ACO	7	200	200	200	0.00	100	17.620

SD = Standard Deviation.

Fig.7. shows the comparative convergence characteristics of the Ant Colony optimization (ACO), Genetic Algorithm (GA), and Tabu Search (TS). Fig.8 shows the distribution outlines of the best solution of each trial. Almost all investment costs obtained by the ACO method are lower. This verifies that the ACO method has a better quality of solution.

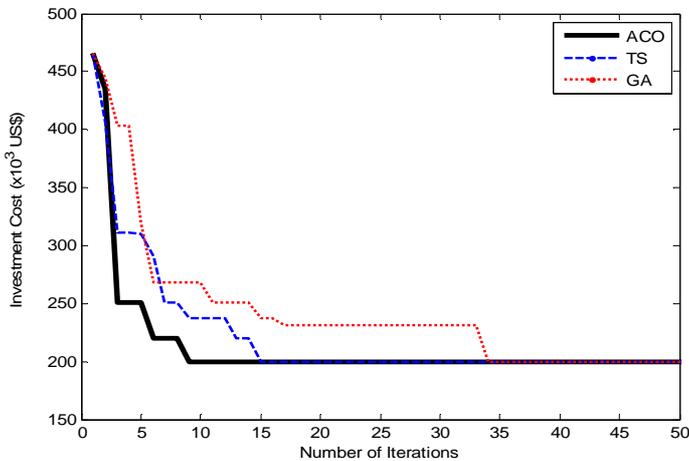


Figure 7. Convergence characteristics of the three methods.

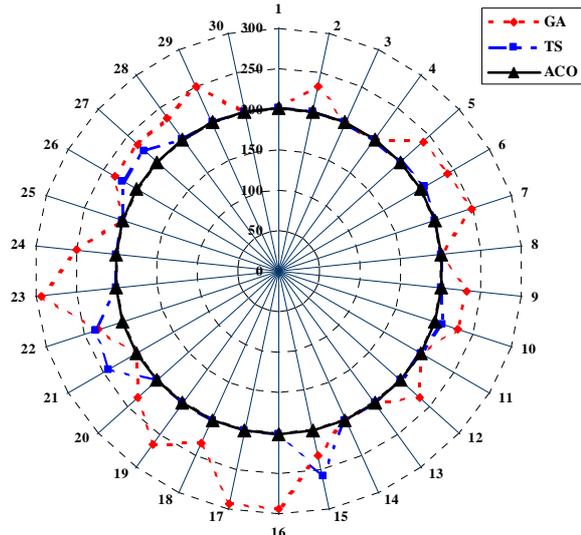


Figure 8. Distributions of investment costs of the three methods.

#### 4. Conclusion

This paper proposes a novel approach for adopting and ACO search algorithm to solve a TEP problem that works corporately with the DC power flow model. The proposed method with Garver's 6-bus test system which gives a good performance in comparison to the conventional GA and TS methods in terms of less calculation time, better quality of solution and more stable-convergence characteristic.

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